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High-Energy Electron Cooling

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Introduction

Electron cooling at high energys with an electron beam circulating in a storage ring was proposed a long time ago,[1] but the idea was dismissed with a premature judgement of the impossibility of achieving a reasonably fast cooling rate with the beam density available. For instance, the present Fermilab[2] scheme has a projected cooling time of 50 msec with an electron current density of 1 A/cm^2 at β = 0.566. At larger energies, because of the strong dependence of the cooling rate on the beam momentum, a reasonable cooling rate can be obtained only with very high electron densities. Recently C. Rubbia[3] pointed out that indeed such large densities are available in stored electron bunches. An average beam current of 100 mA already would correspond to a peak current of tens of amperes. The beam transverse size can be made quite small, down to a millimeter or even less, giving a local density of thousands of A/cm^2 or more.

Rubbia's second point was that at high energies, electrons radiate, so whatever momentum is transferred to them by cooling a proton or antiproton beam will be carried away as radiation, allowing the electron beam to preserve its size, though at the cost of some enlargement.

Finally, the third thing pointed out by Rubbia is that at high energies fast cooling rates are not necessarily required.

There are too possible applications of the high-energy electron cooling:

1. It could be possible to raise the beam-beam limit from the canonical number of $\Delta\nu$ = 0.005 to, say, $\Delta\nu$ = 0.02. This would increase the luminosity by an order of magnitude. Indeed

larger $\Delta\nu$ values cause shortening of the beam lifetime because of a hypothetical Arnol'd diffusion process. The effects of this process can eventually be balanced with electron cooling.

2. The one-beam lifetime itself, even in the absence of the second one, could be too small due to processes like gas scattering. The "heating" of the proton beam caused by such process could then be balanced off by taking the "heat" away from the beam by means of "electron cooling".

In the following we shall look in more detail at the feasibility of high-energy electron cooling, especially in the context of an experiment for the Main Ring with the aim of lengthening the beam lifetime. Although some approximation in our approach cannot be avoided, we are nevertheless mostly interested in a self-consistent solution which takes into account the behavior of the equilibrium of the proton (antiproton) beam as well as the electron beam, which we assume is circulating in a storage ring.

At the end, we also look at the features of the electron storage ring which, as one would expect, is mostly made of wiggler magnets.

<u>The Electron and Proton Beams in Absence of Cooling</u>

The high-energy electron cooling scheme is the one outlined in Figure 1. There are two rings: one could be identified with the Main Ring where protons are circulating at a constant energy E_p and the other with an electron storage ring at energy E_e. The two energies are adjusted so that the two beams have the same velocity. The two rings also share a long straight section of length ℓ where proton bunches and electron bunches travel together in the same direction. We make the obvious assumption

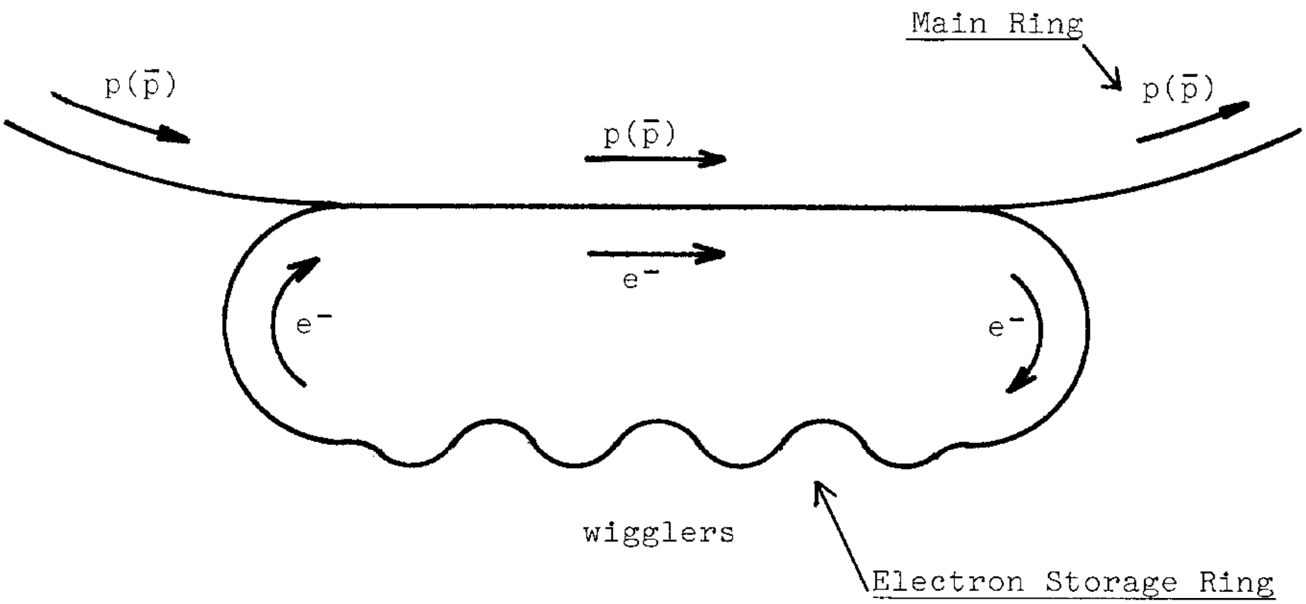

Figure 1. High Energy Electron Cooling Plan

that the two kinds of bunches are roughly matched in size and length.

In the following we shall denote by subscripts "e" and "p" the quantities which refer respectively to the electrons and to the protons.

In the absence of interactions between the two beams, we can write the following equations for the rms beam emittance ($\varepsilon = \sigma^2/\beta$)

$$\frac{d\,\varepsilon_p}{dt} = D_p \tag{1}$$

$$\frac{d\,\varepsilon_e}{dt} = -\frac{2}{\tau}\,\varepsilon_e + D_e. \tag{2}$$

We assume both beams are round, namely, that they have the same horizontal and vertical emittance.

In the absence of diffusion-like processes and of damping effects, the emittances are normally considered invariants. The diffusion coefficient D_p on the right hand side of (1) is primarily given by gas scattering and similar effects. This diffusion is not compensated by damping and will cause a linear increase of the beam emittance with time. The beam-size increase will stop when the beam edge has reached an aperture limitation; after that particles will be continuously lost. In observations in the Main Ring, the following was found[4]

$$D_p = 5\,\frac{P_{torr}}{p^2_{GeV/c}}\ \text{m/sec.}$$

At 100 GeV, with a pressure of about $5\text{x}10^{-8}$ torr, this would correspond to $D_p = 0.25\text{x}10^{-10}$ m/sec.

In eq. (2), τ is the synchrotron radiation-damping time and D_e the quantum-fluctuation diffusion coefficient. The electron beam would have an equilibrium emittance which is given by

$$\bar{\varepsilon}_e = \frac{1}{2}\tau D_e. \qquad (2)$$

This equilibrium value is reached in the e-folding time $\tau/2$.

Observe that τ and D_e depend strongly not only on the beam energy but also on the electron-beam storage ring lattice.[5]

The Electron-Cooling Effect

We want now to modify eqs. (1) and (2) to include the beam-beam interaction, which is supposed to lead to "cooling" of the proton beam at the cost of some "heating" of the electron beam.

Because of the large energy and since the electron beam is already focused by the lattice quadrupoles and rf cavities, we do not have to take into account space-charge effects on the trajectory of the electrons, and we do not have to guide their motion with a solenoid as is done at lower energies. In addition, one can easily verify that at larger energies

$$\theta_{\parallel} << \gamma\, \theta_{\perp},$$

where $\theta_{\parallel}$ and $\theta_{\perp}$ are respectively the longitudinal and transverse relative momentum spreads. This is true for both beams. Thus we are in the situation of a longitudinal flattened ellipsoidal distribution of velocities. In this case, the transverse-energy exchange between the two beams depends only on the transverse emittance of both beams and, therefore, can be decoupled from the longitudinal-energy exchange. In this approximation, the usual formula for the damping rate of the transverse velocity is[6]

$$\frac{1}{\tau_p} = \frac{3\pi\ e^4 L}{m_p m_e\ c^4}\ \frac{\eta_p}{\beta^4\gamma^5}\ \frac{I_e/e}{a_e^{\ 2}\ \theta_e^{\ 3}}\ , \tag{3}$$

where m is the rest mass of a particle, L is the Coulomb logarithm, η_p is the ratio ℓ/C_p, where C_p is the proton ring circumference, the fraction of the circumference over which cooling takes place, I_e is the electron beam current within the bunch, and a_e is the electron beam radius. We are assuming here that beam bunches are cylindrical in shape with uniform particle distributions.

Equation (3) applies to the case of uniform velocity distribution within the electron beam ellipsoid and for proton transverse velocity less than the transverse velocity spread of the electron beam. For the other case, $\theta_{\perp e}$ at the denominator of the right hand side of (3) should eventually be replaced with $\theta_{\perp p}$. To represent a more realistic distribution function with slopes, we shall replace

$$\theta^3_{\ e} \rightarrow (\theta^2_{\ e} + \theta^2_{\ p})^{3/2} \tag{4}$$

in the denominator of the right hand side (3). One should then also introduce a factor $\lesssim 1$ which depends on the distribution. Since this factor is not much different from unity, it will be neglected in the following.

An expression similar to (3), combined with (4), applies also for the electron beam, provided τ_p is replaced with τ_e, m_p with m_e, but not vice versa, and η_p, I_e and a_e are replaced respectively with η_e, I_p and a_p. Since the electron storage ring is smaller than the proton ring, and the lengths of the rings are chosen to synchronize the trasversals of bunches, the ratio η_e/η_p is given by the ratio of the number of proton bunches to the number of electron bunches.

We shall also assume that along the common straight section the β-values of the two rings are constant and we denote them with β_e^* and β_p^*. From the definition of emittance (square of rms beam size/β^*) then we have

$$a^2 = \varepsilon\beta^* \quad \text{and} \quad \theta^2 = \varepsilon/\beta^*, \tag{5}$$

which we can use in the right hand side of (3).

Disregarding any other processes than the interaction between the two beams, the emittance equations are

$$\frac{d\varepsilon_p}{dt} = -\frac{2}{\tau_p}\left(\varepsilon_p - \frac{m_e}{m_p}\varepsilon_e\right) \tag{6}$$

$$\frac{d\varepsilon_e}{dt} = -\frac{2}{\tau_e}\left(\varepsilon_e - \frac{m_p}{m_e}\varepsilon_p\right) \tag{7}$$

where τ_p is given by (3) combined with (4) and (5) and τ_e by a similar derivation. Equations (6) and (7) are equivalent to the energy exchange between two gases put in contact at different temperatures. Equilibrium is reached when the two temperatures are equal. In our case the beam temperature is given by $m\varepsilon$. The times τ_p and τ_e are equivalent to the relaxation times to reach equilibrium.

Observe that in terms of temperature, the relaxation times for the two beams would be the same, but in terms of emittances as shown by (6) and (7) the dependence on the masses is

$$\tau_p \sim m_p m_e \quad \text{and} \quad \tau_e \sim m_e^{\ 2}.$$

Thus the electron beam "heating" time is at least 2000 times smaller than the proton beam "cooling" time.

When (3), (4) and (5) are combined together, they show that τ_p and τ_e depend on the beam emittances ε_e and ε_p.

Self-Consistent Solution at Equilibrium for Both Beams

Let us now combine Eqs. (1) and (2) with (6) and (7). We obtain

$$\frac{d\varepsilon_p}{dt} = D_p - \frac{2}{\tau_p}\left(\varepsilon_p - \frac{m_e}{m_p}\varepsilon_e\right) \tag{8}$$

$$\frac{d\varepsilon_e}{dt} = D_e - \frac{2}{\tau}\varepsilon_e - \frac{2}{\tau_e}\left(\varepsilon_e - \frac{m_p}{m_e}\varepsilon_p\right). \tag{9}$$

The solution of these equations will determine ε_e and ε_p as function of time. Their equilibrium, asymptotic values $\varepsilon_{p\infty}$, $\varepsilon_{e\infty}$ are calculated by setting the right hand side of eqs. (8) and (9) equal to zero.

Let us rewrite (8) and (9) by putting the dependence of ε_e and ε_p more explicitly

$$\frac{d\varepsilon_p}{dt} = D_p - \kappa_p \frac{\varepsilon_p - \frac{m_e}{m_p}\varepsilon_e}{\varepsilon_e\left(\frac{\varepsilon_e}{\beta_e^*} + \frac{\varepsilon_\rho}{\beta_p^*}\right)^{3/2}} \tag{10}$$

$$\frac{d\varepsilon_e}{dt} = D_e - \frac{2}{\tau}\varepsilon_e - \kappa_e \frac{\varepsilon_e - \frac{m_p}{m_e}\varepsilon_p}{\varepsilon_p\left(\frac{\varepsilon_e}{\beta_e^*} + \frac{\varepsilon_p}{\beta_p^*}\right)^{3/2}}\ , \tag{11}$$

where

$$\kappa_p = \frac{6\pi e^3 L\ \eta_p I_e}{m_p m_e c^4 \beta^4 \gamma^5 \beta_e^*} \tag{12}$$

and

$$\kappa_e = \frac{6\pi e^3 L\ \eta_e I_p}{m_e^2 c^4 \beta^4 \gamma^5 \beta_p^*} . \tag{13}$$

At equilibrium we have

$$\varepsilon_p = \frac{\varepsilon_o \varepsilon_e}{\varepsilon_e - \bar{\varepsilon}_e} , \tag{14}$$

where $\bar{\varepsilon}_e$ is given by Eq. (2') and

$$\varepsilon_o = \frac{1}{2}(\tau \frac{\kappa_e}{\kappa_p} \frac{m_p}{m_e}) D_p = \frac{1}{2} \tau_o D_p . \tag{15}$$

It is reasonable to assume that at equilibrium $\varepsilon_e >> \bar{\varepsilon}_e$; then the proton beam emittance is given by (15) and τ_o would represent the proton-beam "cooling" time near equilibrium.

From (12) and (13) we derive

$$\tau_o = \tau (\frac{m_p}{m_e})^2 \frac{\eta_e}{\eta_p} \frac{\beta_e^*}{\beta_p^*} \frac{I_p}{I_e} . \tag{16}$$

Observe the factor $(m_p/m_e)^2$, which is quite crucial for our analysis: one power of the ratio enters because the ratio of proton time τ_p to the electron time τ_e is proportional to m_p/m_e, and the second power comes from the last term on the right hand side of (11), which represents heating of the electron beam, which must be coped with by synchrotron-radiation damping (τ).

The balance equations (10) and (11) apply in the case that the two beams are matched in size and velocity spread (at least approximately). If one wants to fulfill this condition, then $\varepsilon_e \sim \varepsilon_p$ and $\beta_e^* \sim \beta_p^* = \beta^*$. If one also observes that $m_p \varepsilon_p >> m_e \varepsilon_e$ (that is, the proton beam is always "hotter" than the electron beam) then at equilibrium the electron beam emittance is given by

$$\varepsilon_e = \frac{\kappa_p}{D_p} \frac{\beta^{*3/2}}{\varepsilon_o^{1/2}} . \tag{17}$$

<u>Application to the Main Ring and CERN-SPS</u>

Let us consider the example of the Main Ring at 100 GeV. The electron-beam energy is then 50 MeV. The proton-beam emittance, before gas scattering starts to dilute it, is

$$\varepsilon_p = 2.2\text{x}10^{-8} \text{ m} \tag{18}$$

and the diffusion coefficient

$$D_p = 0.25\text{x}10^{-10} \text{ m/sec.}$$

If we want to "cool" the beam so that it preserves its initial emittance, then the cooling time required from eq. (15) is

$$\boxed{\tau_o = 1.76\text{x}10^3 \text{ sec}} \; . \tag{19}$$

From (17), setting $\varepsilon_e = \varepsilon_o$ and taking $\beta^* \sim 70$ m, as it is in the present Main Ring medium or long straight-section, we derive

$$\boxed{\kappa_p = 1.4\text{x}10^{-25} \text{ m/sec.}} \quad . \tag{20}$$

Let us take ℓ = 10 m for the interaction length; then $\eta_p \simeq 1.6\text{x}10^{-3}$. In addition, L = 15. Then we derive from (12) and (20)

$$\boxed{I_e \simeq 8 \text{ A},} \quad , \tag{21}$$

after having assumed $\beta_e^* \sim \beta_p^* \sim 70$ m. The above is the peak current within the electron bunch. It is a reasonable number.

With 10^{10} protons per bunch, the peak current in the Main Ring is about 1 A.

Let us assume that the number of proton bunches equals the number of electron bunches properly synchronized, so that

$$\eta_e/\eta_p = 1.$$

Then we derive from (16) and (19) the required radiation-damping time

$$\boxed{\tau = 4.4 \text{ msec}} \quad . \tag{22}$$

This number is rather small.

The same calculation could be repeated for the CERN-SPS. Here it seems that D_p is an order of magnitude smaller, because of better vacuum.[7] If all the other parameters remain unchanged, as effectively they are, then the required radiation damping time is also an order of magnitude larger, say around 40-50 msec.

One can repeat the same calculation for larger proton energies, say 200 GeV rather than 100 GeV. If one adopts the same procedure, which is to "freeze" the proton-beam emittance to its invariant value, then

$$\varepsilon_o \sim 1/p \qquad \text{(p, beam momentum)}$$

and presumably

$$D_p \sim 1/p^2 .$$

From (15) then

$$\tau_o \sim p$$

whereas from (17) (with $\varepsilon_e \sim \varepsilon_o$)

$$\kappa_p \sim 1/p^{7/2} .$$

From

$$I_e \sim p^{3/2},$$

and, in conclusion, leaving I_p unchanged, from (16), we derive that the required radiation-damping time increases with the beam momentum as

$$\tau \sim p^{5/2} . \qquad (23)$$

Thus, at 200 GeV, for instance, τ = 25 msec. At the same time the electron-beam energy also increases and reaching the required damping time is easier. Thus this scheme is better at higher energy.

The Electron Storage Ring

In order to achieve a reasonable radiation-damping time at low electron energy, wiggler magnets have to be inserted in the electron ring.

Let us consider the case of E_p = 100 GeV which would correspond to E_e = 50 MeV.

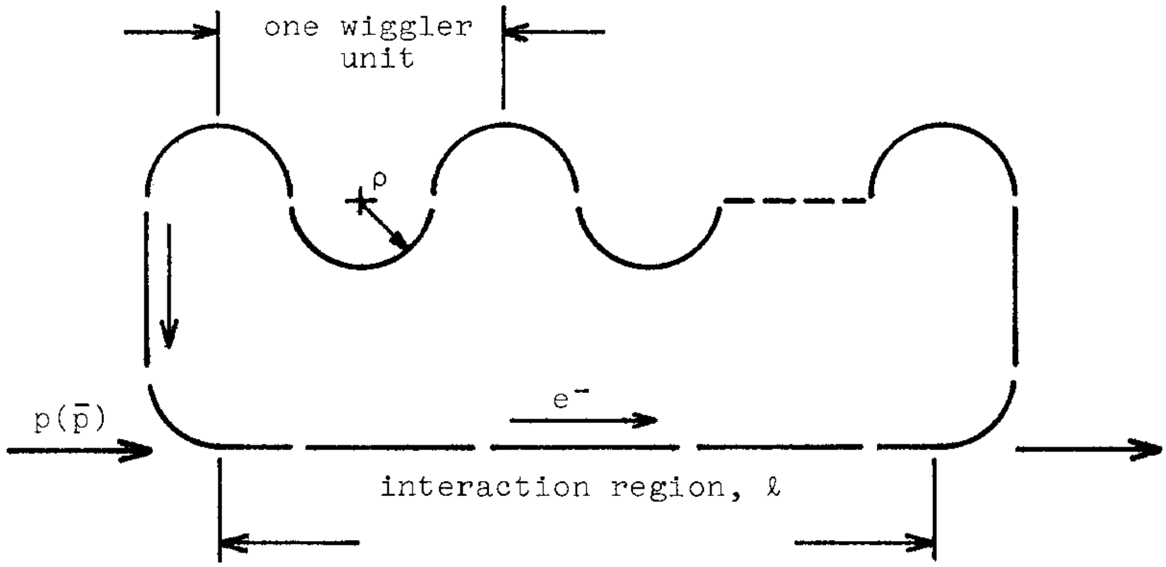

Figure 2. Electron Storage Ring and Wiggler

The electron storage ring could have the shape shown in Fig. 2. Let us define one wiggler unit as the combination of magnets that gives a total bending angle of 2π and let us assume that there are n such units.

The radiation damping time is

$$\tau = T_e \frac{E_e}{U_e} \tag{24}$$

where T_e is the revolution period and

$$U_e = 88.5 \frac{E^4_{e(\text{GeV})}}{\rho_{e(\text{m})}} \eta \quad \text{keV/turn} \tag{25}$$

is the energy loss per revolution, ρ_e being the bending radius in the wiggler magnets. The magnetic rigidity of the electrons at 50 MeV is 1.67 kG·m; therefore, if we take a bending field of 10 kG, which might already be too large for wigglers, then we have

$$\rho_e = 0.167 \text{ m.}$$

From (23)

$$U_e = 3.3 \, \eta \quad \text{eV/turn.}$$

As is shown in Fig. 2 the circumferential length of the electron storage ring will be mostly determined by the space required for the wiggler magnets. We can write

$$C_e \approx 2\text{x}2\pi \; \rho_e \; \eta$$

$$T_e = \frac{C_e}{c} = \frac{4\pi \; \rho_e \; \eta}{c} \; .$$

Inserting these expressions in eq. (22), we find that the radiation-damping time is independent of the number of wigglers. The result is that the radiation damping time cannot be smaller than 100 msec, twenty times more than what is required (eq. (20)) for Fermilab, but only two times larger than what is required for CERN.

If one takes

$$C_e \sim 30 \text{ m}$$

then one would require about 14-15 wigglers.

If the proton-beam momentum p is increased, then obviously the electron-beam momentum must also increase. Then one has the following dependence on the momentum p

$$\rho_e \sim p$$

$$U_e \sim p^3$$

$$T_e \sim p^3,$$

which gives

$$\tau \sim 1/p. \tag{26}$$

The radiation-damping time reduces only linearly by increasing the momentum of the proton beam. In addition, the number n of wigglers for the same storage ring circumference C_e would decrease as 1/p. At the same time, the required damping time versus beam-beam momentum is given by (21).

For the Main Ring at Fermilab, a balance between the required damping time (21) and the damping time that can be achieved (24) for an electron storage-ring circumference of 30 m is reached at E_p = 250 GeV, which corresponds to E_e = 125 MeV. The damping time is about 40 ms and about six wigglers are required.

Thus, in conclusion, the project looks feasible.

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